Splash Fall 2016: Preparing for Quantum Mechanics

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Below is a short list of the topics (mostly from mathematics) that I would like everyone to be comfortable with coming into either of my Splash classes on quantum mechanics. If you see a term you've heard before but aren't sure what it *really* means in a formal sense, don't worry—just look up the term and try to develop an intuitive picture of the concept. In any case, feel free to email me at djg2182@columbia.edu.

Contents

1 QM I: A Mathematical Perspective

In this class, we're going to explore what linear algebra looks like when it's done in an infinite-dimensional space of functions. To that end, it's probably a good idea to have a solid foundation in linear algebra, as well as some basic calculus. Sections marked (*) are optional; any respectable class on linear algebra will devote lots of attention to these topics, but they won't be crucial to our discussion in class.

1.1 Linear Algebra: Vector Spaces and Linear Maps

- Vectors and vector spaces: definitions
- Linear combination a set of vectors
- Linear independence and span
- Basis and dimension of a vector space
- Definition of a linear function or map
- Matrices as a way to represent linear maps (*)
- Kernel, image, rank, nullity, and determinants (*)
- The invertibility of linear maps and matrices
- Transformation of maps under a change of basis (*)

1.2 Linear Algebra: Structure on Vector Spaces

- Definitions: eigenvector, eigenvalue, eigenspace
- Diagonal and diagonalizable matrices (*)
- Various properties of eigenvectors (*)
- The norm and inner product of vectors
- Orthogonality and orthogonal complements (*)
- The existence of orthonormal bases (*)

1.3 Linear Algebra: The Spectral Theorem

- Complex conjugates and matrix transposes; adjoints
- Self-adjoint (Hermitian/symmetric) matrices
- Unitary (orthogonal/norm-preserving) matrices

- The Spectral Theorem: self-adjoint maps have an orthonormal eigenbasis (!!!)
- Normal and commuting maps; simultaneous eigendecomposition (!!!)

1.4 Other Tools from Calculus and Physics

- Facility with complex numbers, including complex exponentials
- Basic differentiation and integration techniques; especially the product and chain rules, and integration by parts. Anything equivalent to a Calculus I course will do, but the more experience with calculus the better.
- From physics: bring an open mind, and not much else. Having a vague understanding of waves may help, but it really won't be of much use.
- Previous experience in quantum mechanics is always welcome. This class is less about quantum mechanics itself than how to think about the structure of the theory, so knowing lots of physics may not be much help.

2 QM II: A Physical Perspective

Ironically, the physical class on quantum mechanics requires a much more extensive mathematical (and physical!) background than the mathematical class. Linear algebra will not be as central to this class; instead, multivariable calculus and some differential equations will give life to the physics. Our "derivation" of the Schrödinger equation essentially starts with the wave equation, so having a decent bit of classical physics under your belt will also be useful. Starred topics are optional, and denote topics that either (a) are of minor importance, or (b) will be covered very quickly in class.

2.1 Calculus and Differential Equations

- Single-variable calculus: differentiation and integration, again with emphasis on the product and chain rules, as well as integration by parts.
- Partial derivatives; div, grad, curl, and the Laplacian
- Multivariable integration and Stokes's, Gauss's, and Green's theorems (*)
- General facility with vector calculus in 3 dimensions
- Basic ODEs: Separation of variables and separable solutions (*)
- Basic ODEs: e.g. solving simple equations like $y'' = \pm ky$

2.2 Classical Physics

- Basic Newtonian mechanics
- Maxwell's equations (in free space)
- The wave equation; plane wave solutions (*)
- The propagation of waves in a cavity (*)
- The photoelectric effect
- Bonus: special relativity! While quantum mechanics is a non-relativistic theory, if there is time at the end of class, we will derive the Klein-Gordon equation, which represents an important first step towards relativistic quantum field theory. (*)